**Problem Set 3**

*Due Wednesday, October 2, 2013 before 9:31am*

*Instructions. This is a GROUP assignment. Please form yourselves into groups of no more than THREE students.*

*You might find Question 6 to be time-consuming. I suggest that you start this question before Tuesday night.*

*Our excellent Teaching Assistant Sicheng GUO suggests that you submit your computations in the form of an Excel file or Python Notebook via Canvas. Because you can enter text into these files, add labels to the file, format it for readability, etc., I think it makes sense put your entire solution into the Excel or Python file, and submit just a single file. I also add the following suggestions:*

*(a) The names of the group members should be clearly marked in your submissions*

*(b) Please make sure that your file is well-organized and clear, with appropriate text, explanations, and formatting.* ***If Mr. Guo cannot figure out what you did, then what you did is wrong.***

1. (Confidence intervals) The spreadsheet PS4\_Q1\_data distributed on the Canvas site includes annual returns on four indices and stocks from 1994-2023, inclusive. **For each data set perform the following:**
2. What is the average annual return over that period? What is the sample standard deviation of the annual returns?
3. If you somewhat bravely assume that the expected return *m* and standard deviation of returns *s* were both constant over the period, then the average return is an estimator of the expected return *m*. Using ideas that I hope you have learned in IE 522, use the average return and standard deviation to form a 95% confidence interval for the expected return *m*.

*Hint*. Remember the distinction between the standard deviation and the standard error. You might first need to convert the standard deviation to the standard error.

1. (Autocorrelations/Independence) The spreadsheet ‘PS4\_Q2\_template’ in the tab contains daily returns for the Hang Seng Index from Sep 2019 – Sep 2024 (this can be obtained from Yahoo finance or other databases). It is possible to test for returns being independent and identically distributed (i.i.d) using autocorrelations. To carry out this test we must first estimate the autocorrelations corr*(rt,rt-)*.
2. Estimate the autocorrelations, , of daily returns (***starting from 10/23/2019 giving an n of 1214 for each autocorrelation***) at different lags, t, from one until 15 days. Which lag (t) has the largest autocorrelation, which has the smallest? Try (briefly) to explain your results.
3. Now consider the following functions of returns*: rt2, |rt|* and estimate the autocorrelations (for example corr*(rt2,rt-))*. of these functions. Compare these autocorrelations to those from part a), what do you find?

Under the assumption of returns being i.i.d, the **sample** autocorrelation (r), from a sample of size n, is normally distributed with mean 0 and variance (1-r2)/(n-2), (standard error = √((1-r2)/(n-2))).

1. Using this information, construct a 99% confidence interval for the sample autocorrelation (r1) of returns (with lag = 1) scaled by √(1-r2), i.e

from for your daily returns of the data set above. Based on your calculation of r1, do you think the returns are i.i.d?

1. Using this information, also construct a 99% confidence interval for the sample autocorrelation of **squared returns** (with lag = 1) scaled by √(1-r2), from for your daily returns of your data set from PS4\_Q2. Based on your calculation of the autocorrelation, do you think the returns are i.i.d?

In fact, under the assumption of i.i.d then we can create a new test statistic,

(where *ri* denotes sample autocorrelations of different lags and *n* is the size of the sample.) which is distributed according to a Chi-squared (c2) distribution with degrees of freedom = *k.*

1. (1 point) Construct a 99% confidence interval for Qk from the returns data in PS4 Q2 where n = 1214 and k = 15.
2. Does the estimate of Qk for **returns** fall in this confidence interval?

Does the estimate of Qk for **squared** **returns** fall in this confidence interval?

Does the estimate of Qk for **absolute** **returns** fall in this confidence interval?

1. What do you think about the assumption that returns are i.i.d?
2. Two securities, A and B, have the following joint distribution of monthly returns (in %):

|  |  |  |
| --- | --- | --- |
| Probability | rA | rB |
| 0.1 | -1.0 | 0.15 |
| 0.8 | 0.5 | 0.15 |
| 0.1 | 0.5 | 1.65 |

1. Compute the means, variances and the covariance of returns for the two securities.

1. Plot the feasible mean-standard deviation [m, σ] combinations assuming that the two securities are the only risky investment assets available.

1. Show on a graph (standard deviation on the y-axis and mean on the x-axis) the portfolios that belong to the mean-variance efficient set?

1. Show that security B is mean-variance dominated by security A yet enters into all efficient portfolios but one. Can you explain this?

*Note:* To Mean-Variance dominate means to have a larger mean and a lower variance (or standard deviation).

1. The tab “Returns of various assets” in the Excel spreadsheet “PS4\_Q4” contains the monthly returns of various assets, including the returns on the S&P 500 index and [Fairfield Sentry](https://en.wikipedia.org/wiki/Fairfield_Greenwich_Group#Fairfield_Sentry_Fund), which was a [Bernie Madoff](https://en.wikipedia.org/wiki/Bernie_Madoff) “feeder fund.” In this question I ask you to consider portfolios formed from combining the S&P 500, Fairfield Sentry, and the risk-free asset. The risk-free rate of return is 0.33% (33 basis points) per month.
2. Let *E*[*rFS*] be the expected monthly return on Fairfield Sentry, *E*[*rSP*] be the expected monthly return on the S&P 500 index, *sFS* be the standard deviation of the monthly returns of Fairfield Sentry, *SP* be the standard deviation of the monthly returns of the S&P 500, and *r* be the correlation coefficient between the returns of Fairfield Sentry and the returns of the S&P 500 index. Using the data in the tab “Returns of various assets,” estimate the expected returns *E*[*rFS*] and *E*[*rSP*], the standard deviations *sp* and *sFS*, and the correlation *r*. (Use the historical average returns as estimates of the expected returns, and use all the data from December 1990 to May 2005.)
3. The risk-free rate of return is 0.33% (33 basis points) per month. Graph the combinations of mean and standard deviation of monthly return that can be obtained by combining the S&P 500 and the risk free asset. Graph the combinations of mean and standard deviation that can be obtained by combining Fairfield Sentry and the risk-free asset. (Put both frontiers on the same graph.)
4. A portfolio that has weight *w* in Fairfield Sentry and weight 1 – *w* in the S&P has expected return and standard deviation



where *E*[*rp*] is the expected return on the portfolio and *sp* the standard deviation of the return on the portfolio. Assume that the risk-free interest rate is *rf* = 0.33% or 33 basis points per month. Find the portfolio weight *w* that gives the maximal Sharpe ratio.

*Note*: You can do this using either ideas you learned in first course in calculus (that is, differentiate with respect to *w* to obtain the first and second-order conditions), using the Excel function Solver, or by trial-and-error search.

The Sharpe Ratio is:

1. Graph the efficient combinations of mean and standard deviation that can be obtained by combining Fairfield Sentry, the S&P 500, and the risk-free asset. (You may add this to the graph from part (b).)
2. In 2005 would you have invested in Fairfield Sentry? Briefly explain why.